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Technical Notes

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Additional Description of Laminar Heat Convection in Tube with Uniform Wall Temperature

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Nomenclature

Cx	=	cross average of x, which can be W_{e-r}, $W_{e-\theta}$, W_{e-z}, W_{c-r}, $W_{c-\theta}$, or W_{c-z}
$e_{i,j}$	=	velocity gradient, a second-order tensor
f	=	friction factor
h	=	heat transfer coefficient
$\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_z$	=	unit vector with respect to the r, θ, z direction
Nu	=	Nusselt number
q	=	components of the heat flux vector
$\mathbf{q}$	=	heat flux vector
R	=	radius of the tube
r, θ, z	=	coordinate axes
Sx	=	span-averaged value of x, which can be W_{e-r}, $W_{e-\theta}$, W_{e-z}, W_{c-r}, $W_{c-\theta}$, or W_{c-z}
T	=	temperature
t	=	time
$\mathbf{v}$	=	velocity vector
v_r, v_θ, v_z	=	components of the velocity vector
W_{c-r}	=	velocity's contribution to the convection of q_r, $v_r \frac{\partial q_r}{\partial r} + v_\theta \frac{\partial q_r}{r\partial \theta} + v_z \frac{\partial q_r}{\partial z} - v_\theta \frac{q_\theta}{r}$
$W_{c-\theta}$	=	velocity's contribution to the convection of q_θ, $v_r \frac{\partial q_\theta}{\partial r} + v_\theta \frac{\partial q_\theta}{r\partial \theta} + v_z \frac{\partial q_\theta}{\partial z} + v_\theta \frac{q_r}{r}$
W_{e-r}	=	velocity gradient's contribution to the convection of q_r, $\frac{\partial v_r}{\partial r} q_r + \frac{\partial v_\theta}{\partial r} q_\theta + \frac{\partial v_z}{\partial r} q_z$
W_{e-z}	=	velocity gradient's contribution to the convection of q_z, $\frac{\partial v_r}{\partial z} q_r + \frac{\partial v_\theta}{\partial z} q_\theta + \frac{\partial v_z}{\partial z} q_z$
W_{c-z}	=	velocity's contribution to the convection of q_z, $v_r \frac{\partial q_z}{\partial r} + v_\theta \frac{\partial q_z}{r\partial \theta} + v_z \frac{\partial q_z}{\partial z}$
$W_{e-\theta}$	=	velocity gradient's contribution to the convection of q_θ, $\frac{\partial v_r}{r\partial \theta} q_r + (\frac{\partial v_\theta}{r\partial \theta} - \frac{v_\theta}{r} + \frac{v_r}{r}) q_\theta + \frac{\partial v_z}{r\partial \theta} q_z$
α	=	thermal diffusivity
λ	=	thermal conductivity
∇	=	operator

Subscripts

r, θ, z	=	along the r, θ, z direction, respectively

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I. Introduction

CONVECTIVE heat transfer is important in many branches of science and in the engineering process [1–5]. In most engineering and science applications, convective heat transfer means that heat passes through the surface of a wall when fluid flows over it [6]. Up until now, most analytical studies of convective heat transfer were based on the energy conservation equation.

In the energy conservation equation, the described parameter is temperature. This parameter has no negative value and, in some cases, heat transport is named "passive scalar convection" or "passive scalar advection" [7–10]. The reason the process is considered passive is because temperature is a parameter of state, not process. Such a parameter changes passively only after receiving the heat described by the process parameter heat flux, which is defined by Fourier's law [11]. Although passive scalar convection and passive scalar advection are used, these terms do not mean that temperature is convected or advected in the convection or advection process. These terms denote only the final state, or the results of the heat convection or advection process for every instantaneous time point.

Matter has a physical parameter named thermal diffusivity. Thermal diffusivity is the diffusivity of heat; in the transport process, this heat is the heat flux, $\mathbf{q}$, a process parameter. On the other hand, in the energy conservation equation, thermal diffusivity is used before the Laplacian operator of temperature, which means that a state parameter such as temperature is transported in the diffusion process. In the example given, thermal diffusivity is diffusivity of temperature.

Because in the energy conservation equation the studied parameter is a state parameter, such as temperature, instead of a process parameter, the understanding of convection occurring on a wall surface is still more confusing. For simple laminar fluid flow, the basic conflict is that, for real fluid flow, the relative velocity between the wall surface and the fluid is zero. From the conservation equation, on the wall surface, only molecular diffusion has an effect, but heat can transport through the wall by convection instead of diffusion only in some cases. The situation of convective heat transfer on the wall is such that the velocity is zero, but the velocity gradient (defined as $\nabla \mathbf{v}$, a second-order tensor) exists. One would expect that the velocity gradient would make some contribution to the convection of heat on the wall, but there is no place for the velocity gradient in the energy conservation equation.

These stated facts mean that one would use the energy conservation equation to determine the temperature and, from the temperature, determine the heat transfer flux, but one lacks an additional description necessary to clearly understand this process. In this paper, we show this additional description. As one example of its application, we use this description to show the deeper understanding of convective heat transfer in a laminar fluid flow through a tube with a uniform wall temperature.

II. Description of Convective Heat Transfer and Mass Transfer in Terms of Process Parameters

If the dissipative heat of fluid flow is omitted and the fluid's physical parameters are constant, the conservation of energy, in cylindrical coordinates, can be written as follows:

$$\partial T/\partial t + (\mathbf{v}\cdot\nabla)T = \alpha\nabla^2 T \tag{1}$$

The equation of heat in flux form must obey the law of energy conservation. Based on this consideration, the right starting point from which to obtain an additional description is performing a mathematical operation on Eq. (1). After performing operation ∇ on Eq. (1), multiplying $-\lambda$ with all items of Eq. (1), and considering Fourier's law, we have

$$\partial\mathbf{q}/\partial t + (\mathbf{v}\cdot\nabla)\mathbf{q} + e_{ij}\cdot\mathbf{q} = \alpha\nabla^2\mathbf{q} \tag{2}$$

where $\mathbf{q} = -\lambda\nabla T$ and $\nabla\mathbf{v} = e_{ij}$

Inspecting Eq. (2), we find that $(\mathbf{v}\cdot\nabla)\mathbf{q}$ is the convection term caused by velocity. $e_{ij}\cdot\mathbf{q}$ is the convection term caused by the velocity gradient. The right-hand side of Eq. (2) presents the heat transported by the diffusion process.

Equation (2) can be written as

$$\partial q_r/\partial t + W_{c-r} + W_{e-r} = \alpha\nabla^2 q_r \tag{3}$$

$$\partial q_\theta/\partial t + W_{c-\theta} + W_{e-\theta} = \alpha\nabla^2 q_\theta \tag{4}$$

$$\partial q_z/\partial t + W_{c-z} + W_{e-z} = \alpha\nabla^2 q_z \tag{5}$$

III. Application of the Additional Description

In the following sections, we demonstrate how to use the additional description to understand the convective process in laminar flow through a circular tube with uniform wall temperature.

A. Problem Studied

The steady-state laminar convective heat transfer in a tube is studied here. The main flow is directed along z. Two cases of convective heat transfer are simulated: the developing case and the fully developed case. For developing fluid flow and heat transfer, the length of the tube is $L_z = 16R$, but, for the fully developed case, $L_z = 4R$. The numerical method reported by Patankar [12] and Li and Tao [13] is used to determine every term in Eqs. (3–5). When the grid size for the developing case is $31\times 41\times 200$ and for the fully developed case is $31\times 41\times 60$, grid independent results can be obtained. These grid sizes are sufficient to obtain reliable results for the higher-order difference terms appearing in Eqs. (3–5). For different Reynolds numbers of less than 2000, the characteristics are the same. Here, we only presented the results at $Re = 800$. The numerical method and its uncertainty are tested by existing data in the fully developed region: $Nu = 3.66$ and $fRe = 64$. The relative errors for the numerical results are below 1.0%.

B. Explanations of the Convective Heat Transfer Process Using the Additional Description

1. *In the Developing Region on the Wall Surface*

On the wall surface, the velocity is zero and the convection of heat fluxes corresponding to the three directions caused by the velocity are all zero. These are represented by W_{c-r}, $W_{c-\theta}$, and W_{c-z}; they are all zero. SW_{c-r} on the wall surface is shown in Fig. 1a. The velocity makes no contribution to the convection of q_r. SW_{e-r}, presented in Fig. 1a, indicates that the velocity gradient makes a contribution to this convection. In the inlet region, SW_{e-r} is very large and decreases rapidly along z. The distribution of SW_{e-r} along the main flow corresponds to Sq_r, as shown in Fig. 1c. These figures demonstrate that the larger SW_{e-r}, the stronger convection of q_r occurs on the surface. As shown in Fig. 1, the velocity gradient contributes to this convection. The velocity gradient's contribution to this convection

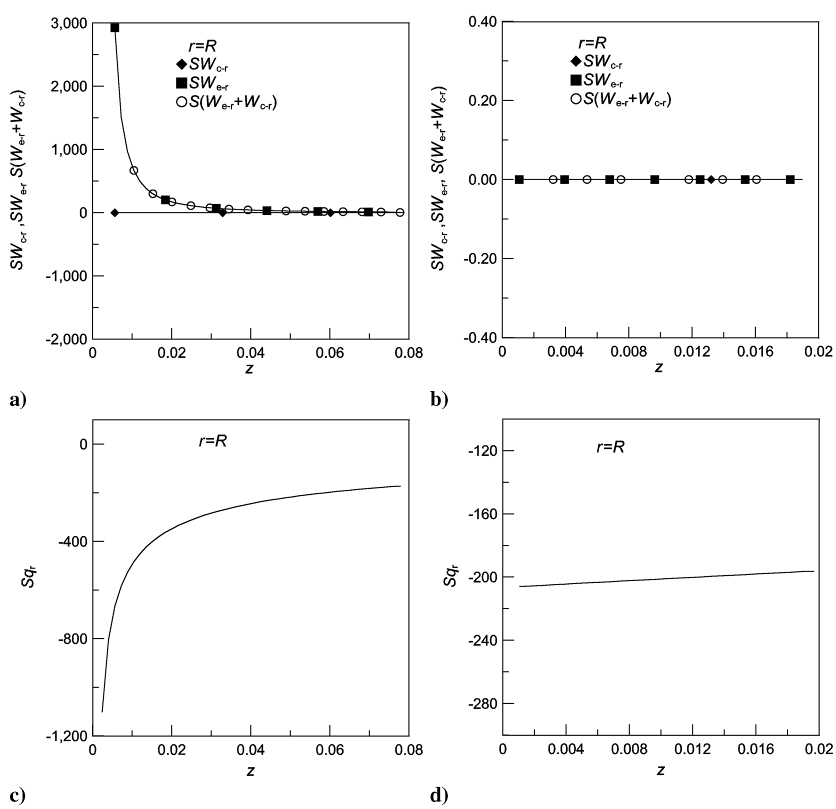

Fig. 1 Span-averaged velocity, velocity gradient on the wall surface: a) span-averaged velocity, velocity gradient terms in the developing region; b) span-averaged heat flux in the normal direction of the wall in the developing region; c) span-averaged velocity, velocity gradient terms in the fully developed region; and d) span-averaged heat flux in the normal direction of the wall in the fully developed region.

comes from $\partial v_r/\partial r$ because, in the developing region, this term exists.

2. In the Developing Region Inside the Flow

q_z quantifies the intensity of the convective heat transfer on the wall surfaces because all of the heat transported from the wall is carried away by the flow. If the convection is strong, the fluid will have a very steeply rising temperature along the main flow. According to Fourier's law, the steeper the temperature rise, the smaller the value of q_z. As shown in Fig. 2a, inside the flow, both the velocity and the velocity gradient make contributions to the convection of q_z, but, unfortunately, the values of CW_{e-z} and CW_{c-z} have different signs. $C(W_{e-z} + W_{c-z})$ shares the sign of CW_{c-z}, which means that the velocity gradient has a negative effect on the transport of q_z. The trend of $C(W_{e-z} + W_{c-z})$ corresponds to the trend of Cq_z, presented in Fig. 2c, but this agreement is not very good. This implies that the diffusion process plays a very important role in the transport of q_z.

The distributions of W_{e-r}, W_{c-r}, and their summation along r at different z positions are presented in Fig. 3. In Fig. 3a, W_{e-r} has a peak value near the wall surface. This peak value decreases downstream. The locations of these peak values move inside the flow downstream. In the center of the tube, W_{e-r} is zero. W_{c-r} is presented in Fig. 3b. Along r, this term has two peak values; one is positive, but the other is negative. The amplitude of the near wall peak is large. This amplitude decreases downstream. Compared with W_{e-r}, W_{c-r} has a larger absolute peak value near the wall surface. W_{e-r} and W_{c-r} have the same sign near the wall surface. That means, in this region, W_{e-r} and W_{c-r} make the same direction contributions to the convection of q_r. The summation of W_{e-r} and W_{c-r} is presented in Fig. 3c. The distribution of q_r, as shown in Fig. 3d, is obtained. A large peak value for this summation does not necessarily correspond to a large peak value for q_r. Every peak value corresponds to a turning point of q_r.

For the transport of q_z along r, more detailed information can be obtained from Fig. 4. At different points along z, W_{e-z} and W_{c-z} have different values, as shown in Figs. 4a and 4b. There is a peak value for W_{e-z}, but there are two peak values for W_{c-z}. On the wall surface, W_{e-z} and W_{c-z} are zero. The amplitude of the peak value of W_{c-z} is 3 times larger than its counterpart of W_{e-z}. Near the wall surface, the summation of W_{e-z} and W_{c-z} has a positive value, as shown in Fig. 4c. Both the velocity and the velocity gradient make different contributions to the transport of q_z. The local heat flux of q_z behaves as shown in Fig. 4d. Every peak point of $W_{e-z} + W_{c-z}$ corresponds to a turning point of q_z. A larger peak value means a steep turning point of the distribution of q_z, as shown in Fig. 4d. Near the wall surface, there is a region with a large absolute value of q_z, especially in the inlet region. A larger absolute value of q_z indicates strong heat transport along the main flow direction. Taking the value of $W_{e-z} + W_{c-z}$ as a reference, the velocity gradient term makes a negative contribution to the transport of q_z because it has a different sign from that of $W_{e-z} + W_{c-z}$. The velocity term has the same sign as that of this summation, so it contributes to the transport of q_z.

3. In the Fully Developed Region on the Wall Surface

When fluid flow and heat transfer are fully developed, on the wall surface, not only W_{c-r} but also W_{e-r} is zero. SW_{c-r} and SW_{e-r} are presented in Fig. 1b. Neither the velocity nor the velocity gradient makes a contribution to the convection of q_r, but one still has a decreased q_r, as shown in Fig. 1d. The transport of q_r only takes the diffusion process on the wall surface.

4. In the Fully Developed Region Inside the Flow

As shown in Fig. 2b, CW_{e-z} is zero but CW_{c-z} is not because $v_z \partial q_z/\partial z$ is just W_{c-z}. Because of the existence of CW_{c-z}, the convection transport of q_z occurs and we have the distribution of q_z, as shown in Fig. 2d. Along the main flow, the absolute value of q_z decreases.

The distributions of W_{e-r} and W_{c-r} are presented in Figs. 5a and 5b. W_{e-r} and W_{c-r} have positive values and, on the surface and at the

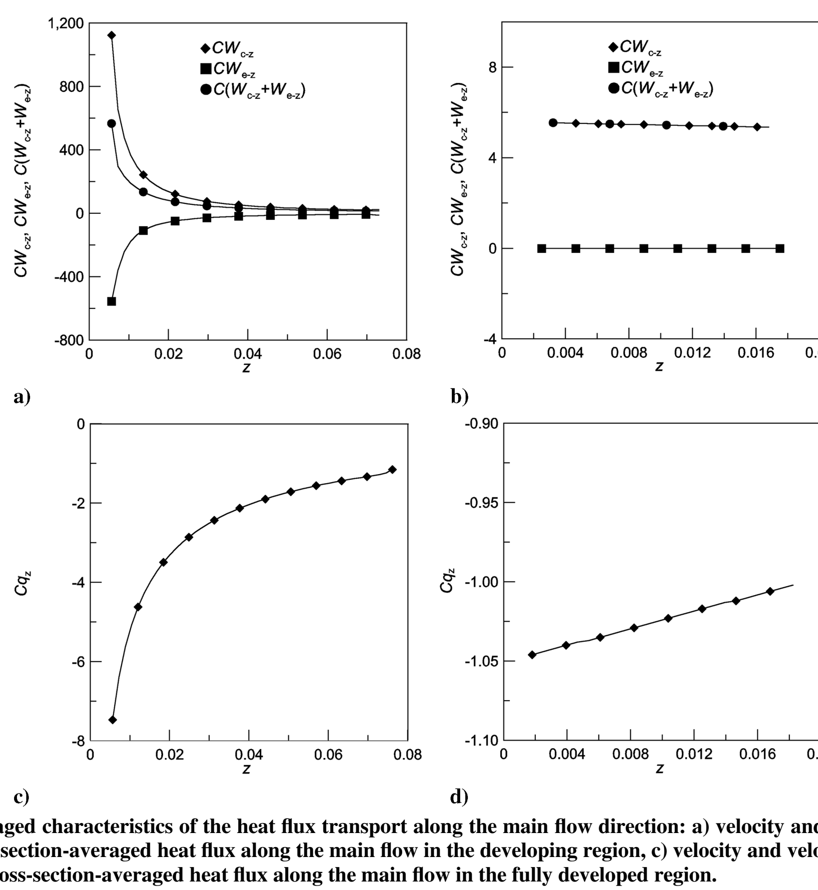

Fig. 2 Cross-section-averaged characteristics of the heat flux transport along the main flow direction: a) velocity and velocity gradient terms in the developing region, b) cross-section-averaged heat flux along the main flow in the developing region, c) velocity and velocity gradient terms in the fully developed region, and d) cross-section-averaged heat flux along the main flow in the fully developed region.

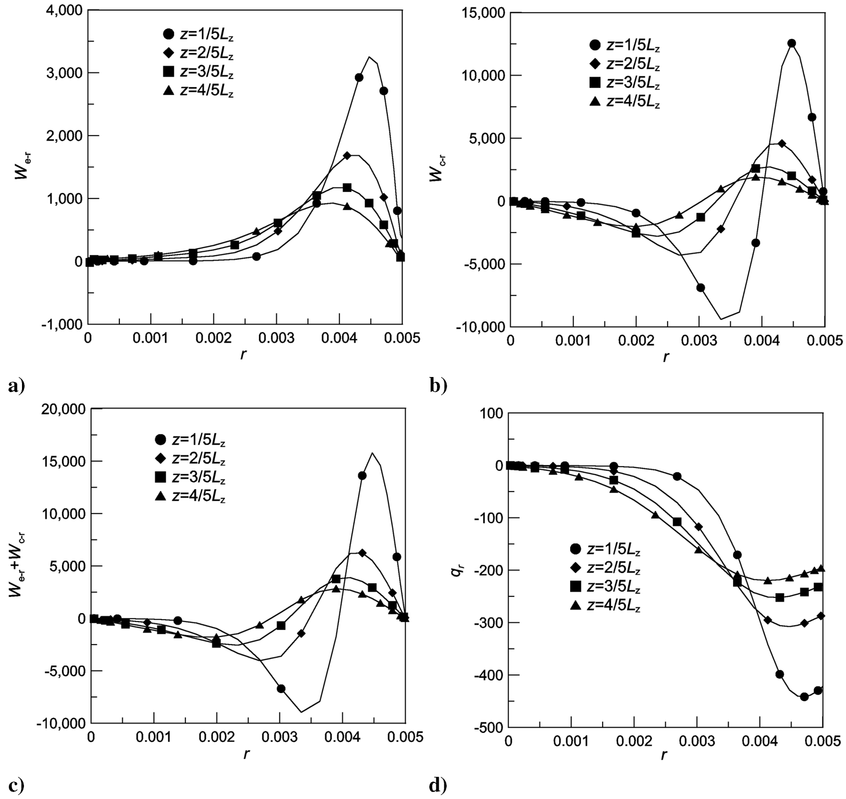

Fig. 3 Radial characteristics of the transport of the heat flux *t* along the *r* direction in the developing region: a) velocity gradient term, b) velocity term, c) summation of the velocity and velocity gradient terms, and d) heat flux in the *r* direction.

center of the tube, they are zero. $W_{e-r} + W_{c-r}$ is presented in Fig. 5c. W_{e-r} and W_{c-r} have the same order values, but the velocity gradient term is comparably small. There are maximum values of W_{e-r} and W_{c-r} around $0.5R$. W_{e-r} has the same sign as W_{c-r}, which means that the velocity and the velocity gradient contribute to the convection of q_r. Based on the values of W_{e-r} and W_{c-r}, the velocity has a slightly larger effect on the convection of q_r than the velocity gradient does. q_r has the distribution shown in Fig. 5d.

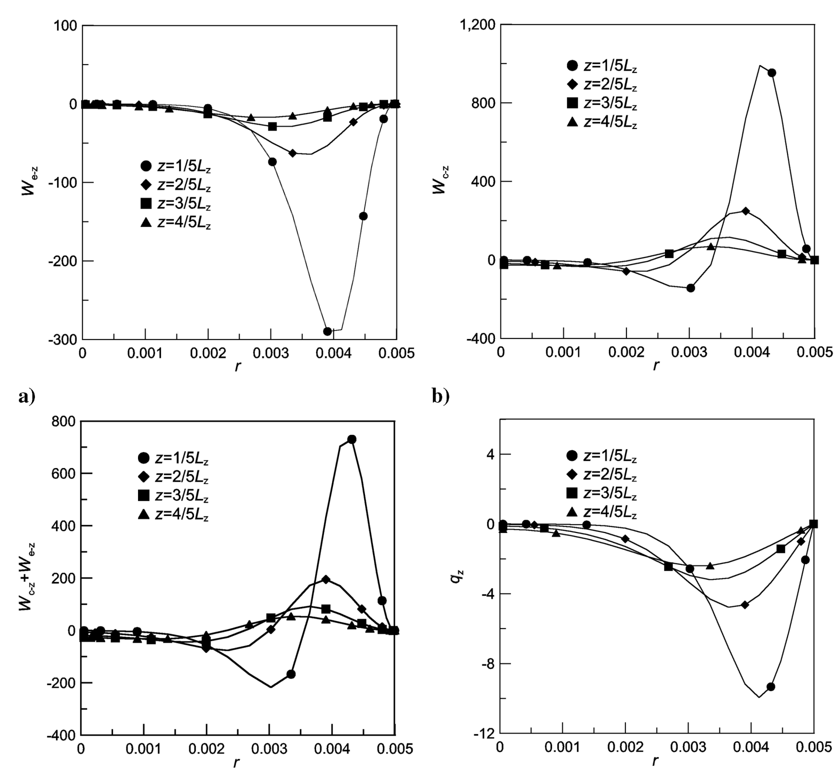

Fig. 4 Radial characteristics of the transport of the heat flux along the main flow direction in the developing region: a) velocity gradient term, b) velocity term, c) summation of the velocity and velocity gradient terms, and d) heat flux along the main flow direction.

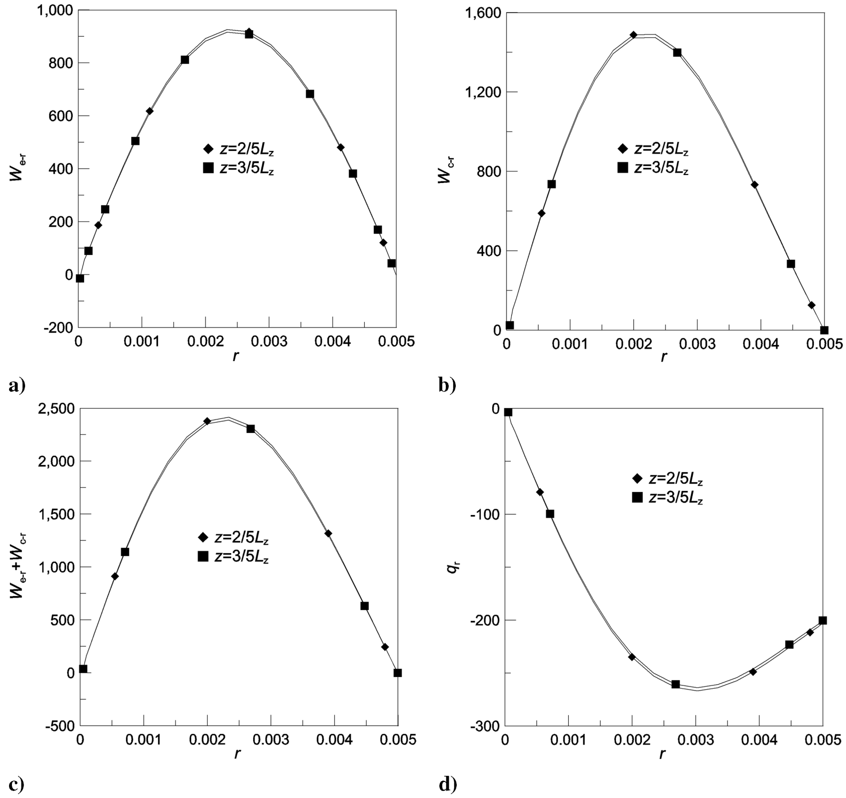

Fig. 5 Radial characteristics of the heat flux transport along the *r* direction in the fully developed region: a) velocity gradient term, b) velocity term, c) summation of the velocity and velocity gradient terms, and d) heat flux in the *r* direction.

$W_{e-z} = 0$, as shown in Fig. 6a. W_{c-z} changes from zero at the surface to a positive value of about 20 at the center of the tube, as shown in Fig. 6b. At the center of the tube, a strong drawing force from the convection of q_z exists. The summation of the velocity and the velocity gradient is shown in Fig. 6c. q_z changes from zero at the surface to a value of about -2.6 at the center of the tube. The trend of W_{c-z} corresponds to q_z in mirror fashion, as illustrated in Fig. 6d. These figures indicate that, in the fully developed region, the velocity

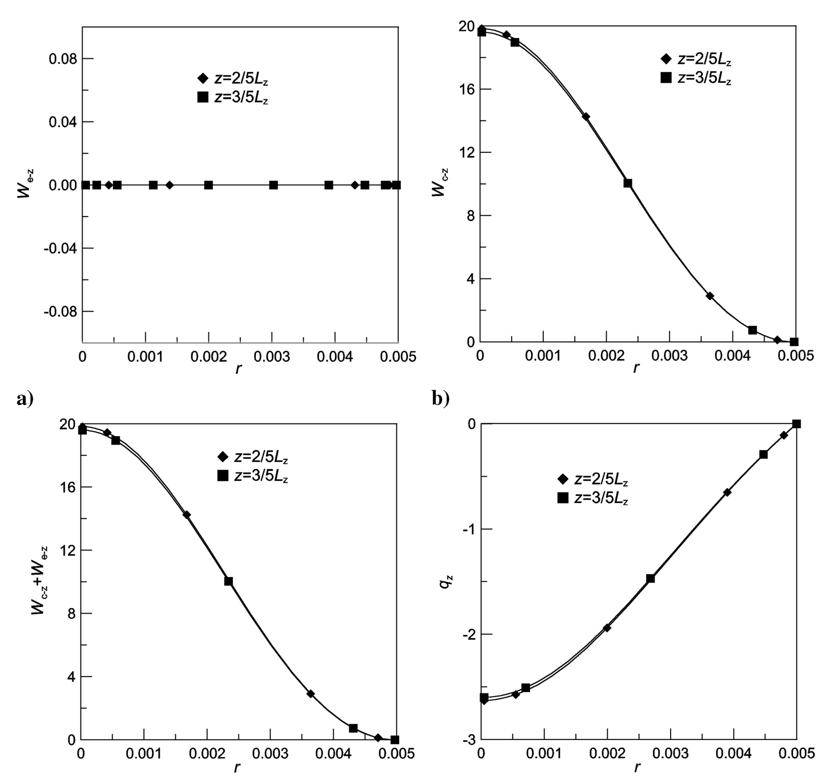

Fig. 6 Radial characteristics of the heat flux transport along the main flow direction in the fully developed region: a) velocity term, b) velocity gradient term, c) summation of the velocity and velocity gradient terms, and d) heat flux along the main flow direction.

gradient makes no contribution to the convection of q_z, which is only powered by velocity along the main flow.

IV. Conclusions

In this paper, we used the process parameter of heat flux as the parameter to describe the convection process and applied the description to understand the process of convective heat transfer in laminar flow through a tube with a uniform wall temperature. The findings reveal that this description can more clearly explain the convective heat transfer process. The significant characteristic of this description is that the contributions that fluid flow make to convection can be divided into the contributions of velocity and its gradient. The deeper understanding of the convection heat transfer process in a flow through a tube with a uniform temperature can be summarized as follows:

1) On the wall surface of the developing region, the velocity gradient contributes to the convection of heat flux normal to the wall surface, but this velocity gradient is not the gradient of the main flow's velocity component. The trend of the velocity gradient term has the same trend as that of the heat flux normal to the wall surface.

2) Inside the developing flow and heat transfer region, both the velocity and the velocity gradient contribute to the convection of heat flux along the main flow direction. But the velocity term has a different sign from the velocity gradient term. Thus, the velocity gradient has a negative effect on this convection.

3) Inside the developing and the fully developed fluid flow and heat transfer region, both the velocity and the velocity gradient contribute to the convection of heat flux in a radial direction. But the velocity term plays the main role in this convection.

4) In the fully developed region on the wall surface, the velocity gradient makes no contribution to the convection of heat flux normal to the wall surface. The heat flux normal to the wall is transported through the wall by diffusion only.

5) In the fully developed region, contributions to the convection of heat flux along the main flow direction come from the main velocity multiplied with the gradient of the heat flux in the main flow direction. The velocity gradient makes no contribution to this convection.

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References

[1] Pasquill, F., and Snith, F. B., *Atmospheric Diffusion*, Ellis Horwood, Chichester, England, U.K., 1983.

[2] Dibble, R. W., Warnatz, J., and Maas, U., *Combustion: Physical and Chemical Fundamentals, Modeling and Simulation, Experiments, Pollutant Formation*, Springer–Verlag, New York, 1996.

[3] Atema, J., "Chemical Signals in the Marine Environment: Dispersal, Detection, and Temporal Signal Analysis," *Proceedings of the National Academy of Sciences of the United States of America*, Vol. 92, No. 1, Jan. 1995, pp. 62–66. doi:10.1073/pnas.92.1.62

[4] Webb, R. L., *Principles of Enhanced Heat Transfer*, Wiley, New York, 1991.

[5] Bergles, A. E., "Recent Developments in Convective Heat Transfer Augmentation," *Applied Mechanics Reviews*, Vol. 26, No. 3, 1973, pp. 675–682.

[6] Kays, W. M., Crawford, M. E., and Weigand, B., *Convection Heat and Mass Transfer*, 4th ed., McGraw–Hill, New York, 2005.

[7] Kraichnan, R. H., "Small-Scale Structure of a Scalar Field Convected by Turbulence," *Physics of Fluids*, Vol. 11, No. 5, May 1968, p. 945. doi:10.1063/1.1692063

[8] Kraichnan, R. H., "Convection of a Passive Scalar by a Quasi-Uniform Random Straining Field," *Journal of Fluid Mechanics*, Vol. 64, No. 4, July 1974, pp. 737–762. doi:10.1017/S0022112074001881

[9] Kraichnan, R. H., "Anomalous Scaling of a Randomly Advected Passive Scalar," *Physical Review Letters*, Vol. 72, No. 7, 1994, pp. 1016–1019. doi:10.1103/PhysRevLett.72.1016

[10] Fairhall, A. O., L'vov, G., and Procaccia, V. I., "Anomalous Scaling in a Model of Passive Scalar Advection," *Physical Review E (Statistical Physics, Plasmas, Fluids, and Related Interdisciplinary Topics)*, Vol. 53, No. 4, 1996, pp. 3518–3535. doi:10.1103/PhysRevE.53.3518

[11] Bird, R. B., Stewart, W. E., and Lightfoot, E. N., *Transport Phenomena*, 2nd ed., Wiley, New York, 1960.

[12] Patankar, S. V., *Numerical Heat Transfer and Fluid Flow*, Hemisphere, Washington, D.C., 1981.

[13] Li, Z. Y., and Tao, W. Q., "A New Stability-Guaranteed Second-Order Difference Scheme," *Numerical Heat Transfer, Part B, Fundamentals*, Vol. 42, No. 4, Oct. 2002, pp. 349–365. doi:10.1080/10407790190053987